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Open-start mathematics problems: an approach to assessing problem solving

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This article describes one type of mathematical problem, open-start problems, and discusses their potential for use in assessment. In open-start problems how one starts to address the problem can vary but they have a correct answer. We argue that the use of open-start problems in assessment could positively influence classroom mathematics teaching. The article provides a brief review of problem solving and describes open-start problems in detail. The article then considers how open-start problems could address some important concerns in the teaching and assessment of mathematics and raises issues with regard to the future use of open-start problems in assessment.

1. Introduction

What is the use of students learning mathematics if they cannot use it to solve problems? So goes up the cry, which is both a call to do more problem solving in schools, and a statement of despair that very often students struggle to solve what appear to be simple problems which relate directly to the mathematics that they learned recently. According to Lester (1994, p. 661), 'most mathematics educators agree that the development of students' problem-solving abilities is a primary objective of instruction'. Unfortunately, as Schoenfeld (1992) observes, problems and problem solving have had multiple and often contradictory meanings through the years, ranging from any task to be

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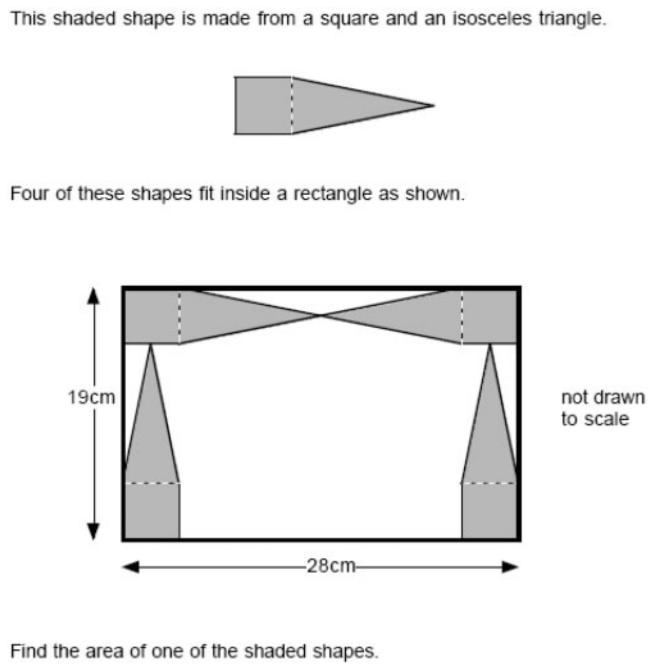

FIG. 1. An open-start problem.

undertaken, to activities requiring the application of particular procedures, to dealing with ‘word problems’, to ‘creative’ thinking and more; the meaning of Lester’s truism clearly has varied.

In this article, we describe and discuss the potential for use in assessment of one type of mathematical problem, which we have named ‘open-start problems’. An example of an open-start problem is given in Fig. 1.

The term ‘open-start’ refers to the fact that how one starts to address the problem can vary. For example, the problem in Fig. 1 can be approached arithmetically, algebraically, geometrically or using a hybrid approach (and various strategies can be used within each of these approaches). Open-start problems are different from many of those classified as open-ended problems in that they have a correct answer (one might say a ‘closed-end’). In the fourfold ‘degree of openness’ classification of test items developed by Hellström *et al*. (2001), based on one/different strategies (starts) and one/different answers (ends), open-start problems would be many-strategies-one-answer test items.

We focus on these problems because: (i) they could contribute towards a solution to an assessment problem in England at the moment; (ii) although our consideration of them is in assessment, we feel that their use in assessment could positively influence classroom mathematics teaching. This article focuses on assessment for 16-year-old students but open-start problems could be designed for elementary school or university assessment.

The remainder of the article is structured as follows:

- First, we provide some background to the presence of problem solving in school-leaving examinations in England.
- We then offer a brief review of problem solving as a process.
- Third, we describe open-start problems in more detail, with examples.

- Next, we look at how open-start problems could address some important concerns in the teaching and assessment of mathematics.
- Finally, we consider issues in the use of open-start problems in assessment.

2. Background

There has been a degree of dissatisfaction with the quality of school mathematics education in England for at least a hundred years. Some of this may be explained as the inevitable result of taking for granted whatever improvements do take place and then asking for more. Periodically, however, governments have conceded that reappraisal is necessary, and have set up an enquiry. Perhaps the most significant of these was the Cockcroft inquiry into the teaching of mathematics in schools (Cockcroft, 1982). In the late 1970s, employers had become increasingly vocal in their dissatisfaction with the inabilities, as they saw them, of school leavers to cope with 'basic' mathematics in the workplace, and to use mathematics to solve problems. The resulting Cockcroft Report was a substantial response to this state of affairs. One of its recommendations, seen as highly significant by those in mathematics education, was paragraph 243, which suggested that all mathematics teaching should include extended pieces of work, investigations and problem solving. The report was a major feature in the reform of school mathematics in England in the 1980s and this reform included the introduction of coursework, since the assessment of extended pieces of work was felt to be impossible in conventional examinations. However, there was some lack of clarity as to the kind of problem solving implied by 'coursework' and the extent to which the kind of mathematical thinking that came with it went beyond the traditional fare of factual recall and memorized algorithms.

Awarding Bodies (non-government organizations which produce syllabuses and examination papers) produced exemplar materials for coursework, together with specimen work from students to support schools. These illustrations, however, demonstrated a very particular way of doing investigations. Many schools understandably tended to imitate this, teaching it to their students so that the students were able to regurgitate the process with their own assessed pieces of coursework. It was not uncommon to see lower attaining students unenthusiastically using very elementary mathematics to plan holidays or furnish bedrooms from glossy catalogues, whilst the higher attaining students set off on open-ended tasks in pure mathematics, not entirely sure of the purpose, only to have them closed down by the teacher as time ran out and a 'result' was needed. Some Awarding Bodies offered examinations where similar problem-solving tasks were attempted in a 3 h period. Old habits die hard, especially when there is a need for reliable assessment and manageability of work on relatively open tasks, and for many (though not all) coursework reduced to what might be termed the 'long-multiplication' of investigations—exercises in producing expected behaviours rather than anything involving actually solving a problem.

Coursework became further standardized when the UK government agency that oversaw school-leaving examinations reduced mathematics coursework to two specific tasks, an investigation and a statistics task. Disenchantment with coursework grew. A referendum on mathematics coursework carried out in England on behalf of the government in 2005 saw an overwhelming rejection of it by mathematics teachers. Although investigations and other types of coursework were meant to introduce problem solving into the mathematics classroom by making them a part of the assessment, the standardization of the process of doing them undermined this good intent. Coursework has now been withdrawn from the mathematics school-leaving examination, the General Certificate of Secondary Education (GCSE), in England.

At the same time as the disenchantment with coursework grew, employers and universities again began to complain about the mathematical abilities of their recruits from schools. This led to another inquiry, the Smith Report of 2004 (Smith, 2004) which recognized, amongst other things, the need for the re-instatement of ‘challenge’ into school mathematics courses. The report recommended a second mathematics GCSE for the ‘more able’. Government agencies appointed two teams of independent contractors to take forward the Smith report and investigate potential ‘pathways in mathematics’ for ages 14–19 years. This was Phase 1 of the Pathways project, and the authors of this article were part of one of those teams. Each team produced reports with their own recommendations, and our recommendation for the second GCSE was that it should be: problem-solving orientated; for all students; and based upon the current GCSE curriculum, i.e. without additional mathematics content. On completion of Phase 1, the Awarding Bodies were invited to bid for Phase 2, to take the ideas of Phase 1 forward into an examination system, and to pilot new assessments based on the Phase 1 recommendations. We worked with the Awarding Body AQA (see http://www.aqa.org.uk/) on this, and ‘open-start’ problems were a part of the work we developed.

3. The assessment of problem solving

Problem solving can be seen as a response to a question for which one does not already know a method by which it can be answered. What is done to arrive at an answer to such a question has to be constructed by that person at that time, and may differ from the way that another person might approach the same ‘problem’. In this vein, problem solving is defined by PISA (2003, p.156) as ‘an individual’s capacity to confront and resolve . . . situations where the solution path is not immediately obvious’. Polya’s (1945, 1954) work on this meaning of problem solving, focusing on the cognitive processes involved when dealing with situations that present such questions, has been very influential, and indeed the PISA assessment framework on problem solving was based on it.

Problem solving in school mathematics can also be thought of in terms of the process aspect of problem solving, and, following Polya, Burton (1984) suggests that there are three phases to it:

1. Entry, feeling one’s way; jotting down observations and thoughts; trying out possible modes of attack.
2. Attack, employing specific strategies, first to gather relevant data, then to act on it to move towards a solution.
3. Review/extension, testing the solution, generalizing, extending the problem.

Burton (1984) also suggests a number of processes that can be learned and deployed as strategies to progress when the solution path is not obvious, including: organize the information systematically; work backwards from the intended outcome; work on parts of the problem separately; and many more.

A helpful way of thinking further about such processes is in terms of the contrast between convergent and divergent thinking (Guildford, 1967; Orton, 1992). In the context of problem solving: convergent thinking is reasoning that narrows the focus, using induction or deduction, or other processes such as specialization, to approach the solution; divergent thinking is the more creative process of widening the focus, using transformations of meanings or interpretations (e.g. through reasoning by analogy, or as a result of insight), and other processes such as generalization, all of which may lead to greater understanding, or to multiple solutions.

Much of mathematics requires convergent thinking, but problem solving may be considered to require in addition a degree of divergent thought. However, this raises difficulties for national assessment. As the contributors to Lesh and Lamon (1993) explore, problem solving, along with other desirable elements of mathematics education that involve divergent thinking, is best assessed using ‘authentic’ assessment approaches, such as extended tasks and observation, but the demands of reliability, secrecy and year on year comparability drive national assessments in the opposite direction—towards timed tests.

The challenges to assessment of problem solving in a timed test arise from the fact that assessment of problem solving requires access to evidence of process, and a timed test does not have the option of observation to provide that evidence. It requires students to produce an extended, usually written, response, involving explanation of both their thinking and the proposed solution(s), to act as the evidence, from which the use of problem-solving processes may be inferred.

However, students often do not produce good clear responses. There is a skill in producing clear responses that many students do not possess. The need for clear responses is noted by assessors, in that aspects of 'communication' are included in most lists of desirable 'process skills', but prescribing that it should be there does not mean that it will be there. Indeed, prescribing good communication skills may compound the problem by making good communication essential, something without which other process capabilities cannot be reliably assessed. Also, since tests usually operate by awarding marks, effective assessment requires a mark-scheme that indicates clearly to markers what marks can be awarded for, and this relies on some degree of anticipation of what the candidate might do, and in problem solving this can be difficult.

The most common form of problem-solving questions are the invitations to divergent thinking that Hellström *et al*. (2001) might classify as many-strategy-many-answer test items, often called 'open-ended problems'—in which a situation is described and a non-specific solution asked for. In mathematics there can be two main kinds of open-ended problems: those which involve a requirement to 'explore' a described piece of mathematics, and those in which a 'real-life' problematic situation is described; and mathematics is needed to offer any kind of solution to it. Such problems pose particular difficulties for assessment because the solutions to open-ended problems, by their nature, should not be predictable. As Polya (1954, p. 154) puts it: 'A wise problem solver does not commit himself to a rigid plan'. Adaptive flexibility (Guildford, 1967) is the hallmark of successful problem solving, and although, 'The final form of the solution may be recorded, yet the changing plans and the arguments for and against them are mostly or entirely forgotten.' (Polya, 1954, p. 154). This means that a successful mark-scheme that anticipates what candidates will write, such as particular steps in the solution process, is almost impossible to create. Markers might then be asked to make judgements about worth in relation to whatever is recorded, but given the difficulty that candidates have in formulating clear responses, the evidence that can be used to reward the problem solving that has occurred is rarely there.

The alternative being proposed in this article is that of open-start problems, questions with a specific required solution, but where the means of arriving at it has not been taught, and so will not be 'obvious' to the candidate. In the terms described above, there is a need for divergent thinking to make progress, as well as a need for convergent thinking to arrive at the answer. Open-start problems are relatively constrained compared with open-ended problems, but cannot be solved without divergent thinking at some point, and so can be used to assess problem solving—and with benefits to the two issues outlined above.

4. 'Open-start' mathematics problems

In this section, we attempt to clarify what is being referred to as an 'open-start' problem.

First, it has a closed ending, in that a single answer (or specific set of answers) is sought, and there will be little doubt in the mind of the problem-solver as to what is being asked for, when the end is reached, and an answer has been found. What is not so clear to the problem-solver is where to start on the solution. For a problem to qualify as this sort, a method should not be cued in by the wording of the question. The challenge for the problem-solver is to assemble, from their existing mathematical understanding and knowledge, a strategy that might lead to the answer. A number of points immediately follow from this:

- The mathematical knowledge needed to solve the problem must already be known securely: this is not about assessing curriculum content—it is about assessing the ability to deploy such knowledge.
- The problem-solver must not be familiar with a similar problem—the essence of 'open-start' is that it is not clear where to start and recall of a similar situation would compromise this.
- It will not be clear at the outset whether the strategy will work, and it will have to be accepted by the problem-solver that further attempts may be needed.

Within a timed test, two advantages that open-start problems bring are (i) that the correct answer is evidence of problem-solving processes having occurred, and (ii) the range of likely approaches can be anticipated to a large extent, even though individual students will have operated with 'adaptive flexibility'. As a result, a mark-scheme can be devised which has a reasonable level of reliability, and even students who are not skilled at showing their thinking or methods can be suitably rewarded for their problem-solving abilities.

The two questions below (Figs 2 and 3) arguably fulfil the above requirements.

It is clear what kind of answer is wanted to the question in Fig. 2, but it is far less clear what to do to work it out. The openness of the start may mean that several routes to a solution are possible; some may be elegant, others pedestrian and time consuming. In a marking scheme for an open-start problem, a correct answer by whatever means earns full credit. It is also noticeable with this question that there is ambiguity as to the area of the curriculum in which the problem is located, as this will depend on how the problem-solver perceives the question (see also the problem in Fig. 1).

A number of strategies might be formulated to solve the problem in Fig. 3. Some examples are:

- The direct method of checking each of the thousand numbers—but this, though 'obvious', is expected to be seen as too time consuming to be selected.
- Partitioning the numbers from 1000 to 2000 into sets of 100 numbers i.e. 1000–1099, 1100–1199, 1200–1299, etc. Examine the first set of numbers with three '1' digits and then deduce that all the others will be similar except for the set of number 1100–1199, which will require closer analysis.

The diagram shows two identical overlapping circles.

Each circle has 4/5 of its area shaded.

What fraction of the diagram is shaded?

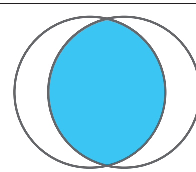

Fig. 2. (This figure appears in colour in the online version of *Teaching Mathematics and its Application*.)

How many integers between 1000 and 2000 have exactly three '1' digits in them?

Fig. 3.

- Treating the problem as one of permutation: the required numbers are all of the form $111x$, where x can occupy any of the last three places in the number and take on any value in the range $2 \leq x \leq 9$ or zero.

The solution paths in each of these overall strategies can also show a great deal of variation, but the answer, 27, is fixed.

5. How open-start problems could address current concerns in England

We would stress that we do not regard the use of open-start problem in mathematics examinations as a universal panacea for the ills of mathematics education; they seem to us to offer a practical means to advance mathematics teaching and assessment in England (and maybe elsewhere) at the time of writing. In this section, we consider timed examinations and the potential of open-start problems to improve classroom teaching. We begin by recalling some of the points made in the sections above.

There is a strong argument that timed written examinations are not really suited for assessing problem solving. Coursework (portfolio work) is arguably much more appropriate as students have time to internalize, and even change, given problems to their own problems; they have time to investigate, generalize and specify. However, given the acute problems perceived to accompany GCSE coursework in England, it appears not to be an option for assessing problem solving in the near and intermediate future.

If not coursework, then what? An examination need not be a timed written examination, it could be open in terms of time or it could be an oral examination. Our focus on timed written examinations reflects our perception that timed written examinations are, at the time of writing, the medium of assessment sought by government agencies; we look for a solution to assessing problem solving that not only has the potential for 'good' but also has the possibility of being adopted.

Even within the constraints of timed written examinations, open-start problems are not the only way to assess problem solving. Open-ended problems have been described above. They have been tried, tested and found to be problematic. They were used, amongst others, by the School Mathematics Project [see Little (1993) for a description]. One hour was given for students to tackle one open-ended problem (a geometric, numeric, algebraic or statistic investigation or a design). Little (then a director at SMP) in 1993, after noting how positively teachers responded, reflected, 'However, prescribing the tasks, prohibiting feedback, and effectively prescribing the responses to the tasks by issuing mark-schemes, all limit the educational worth.' Like Little, we are not drawn to this solution to assessing problem solving.

A perceived problem with mathematics teaching in England is 'a narrow focus on meeting examination requirements by ''teaching to the test'' ' (Ofsted, 2006, p. 3), i.e. only addressing topics, tasks and techniques in the classroom that are likely to be tested in high stakes examinations. Our personal experiences support this as a generalization with exceptions. We do not blame teachers for this. The assessment culture in English education has a political focus on league tables for schools and performance statistics for teachers within schools. This is now so ingrained that it is an exceptional school or teacher (usually exceptional in terms of having highly motivated students) that does not pay significant regard to what is likely to be tested. Without a magic wand to rectify this situation our suggestion is to change the test—make the test less predictable. Open-start problems could provide a means to do this. The open-start problems used in each examination would, thereafter, have to be put aside, not to be used again in an easily recognizable form. So here is a future scenario. A substantial proportion of future GCSE examination questions are open-start questions (the exact proportion to be decided but a sufficient proportion for open-start question to be important for the overall GCSE grade). To prepare students for this examination, to 'teach for the test', teachers must seriously address problem solving in the classroom—not by practising problems that are thought likely to appear, but by instilling an approach to problems of self-reliance—deploying what is known in order to formulate a strategy. This is just starting to be implemented in a pilot examination (AQA, 2008). We do not expect this to be without difficulties but the opportunity to increase the amount of mathematical problem solving by students in classrooms seems too great an opportunity to miss. We now turn to some issues to be addressed.

6. Issues to be addressed

We have described open-start problems and outlined their potential for improving mathematics in school. If these types of problems are taken up, then there is much development work with regard to assessment, to be done. In this final section we sketch issues to be addressed, with the realization that a huge amount of work will lay behind refining each issue in practice.

6.1 Developing problem posers

Open-start problems are not as 'obvious' to write as are more conventional assessment items. Finding a question that students will not know how to do, but which they are nevertheless capable of resolving is a particular talent, and writers of such questions—who we might call 'open-start problem posers'—would need to be developed. Writing an open-start problem, in our experience, is somewhere between an art and a science. Inducting others into this takes time but our experience working with AQA suggests that it can be done.

6.2 Insight

The need to construct a strategy implies a degree of insight—making a connection between pieces of information, recognizing a key feature, being able to recognize the parts that make the whole and so forth. Insight is an elusive phenomenon, though we can be aware of different degrees of it. A recent Secretary of State for Education, faced with a question similar to the one in Fig. 4, responded that there was not enough information for a solution. There is—but insight is required to 'see' how it can be done.

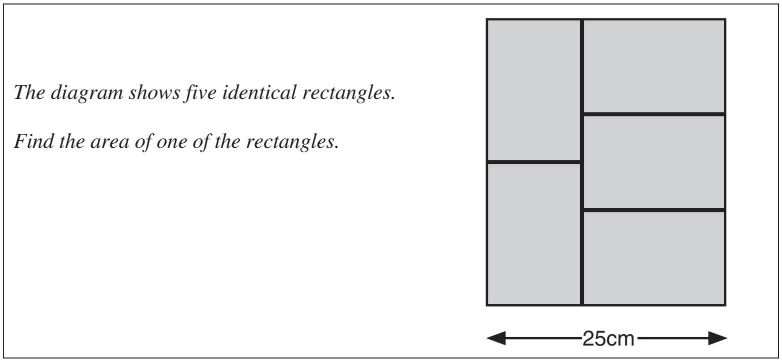

Fig. 4.

A key issue for assessment of this kind of problem solving is how much insight it is reasonable to expect of a school mathematics problem-solver at a particular level at a given age. This appears to be largely uncharted territory at present. People working in the school examination system have accumulated vast experience of what can be expected of students in terms of curriculum knowledge at different ages and achievement levels in respect of conventional mathematics examinations when they are assessments of factual knowledge and recalled procedures. However, expectations around problem solving are less well established and less secure and would need to be developed.

6.3 Item bank

Once an open-start problem has been used in an examination it is effectively 'dead'. Both the reliability of the examination and the prospect of changing classroom practice would be lost if questions were reused, as teachers could try to guess what might come up this year on the basis of what has not been used for a couple of years, and preparing classes to answer specific anticipated questions neither develops problem solving processes nor allows its assessment. However, used questions could form an item bank of open-start problems that teachers could use as examination preparation of the best kind—as a resource for classroom problem solving.

6.4 Levels

What is a problem for some students is not a problem for others. Setting examinations that include problem solving at the lower grades would involve asking questions that some (not least politicians) would need persuading are problems at all, such as the problem in Fig. 5.

There are students who can deal with one of the two constraints (total money and difference in the amounts), but find dealing with both at same time a challenge, and for them this question is a problem—they are not sure how to go about finding a solution, and have to find a strategy and try it out. However, some students will have a developed procedure for dealing with such questions, and for them this is not a problem. Any assessment of problem solving has to ensure that the questions that are set as problems in a particular examination are actually problems for the candidates taking it.

6.5 Mark-schemes

No mark-scheme can circumscribe all possible answers that examination candidates might offer—especially in an item designed to make strategies for a solution not at all obvious. The mark-scheme must offer a sufficiently wide recognition of likely routes to a solution as well as the spirit of interpretation when unexpected strategies appear. A well-written question will minimize the possibility that the correct answer can be come upon by chance, felicitous error or a low-level strategy when a higher one is being sought. This allows, in general, the confident award of full marks for a correct answer without evidence of working. This is useful where a question depends upon a powerful insight, where once the insight is gained, any mathematical calculation may be sufficiently trivial to be done

Divide £5 between two people so that one person gets two pounds more than the other.

FIG. 5.

mentally, since there is then no evidence of problem-solving process written as 'working'. 'Insight', by its nature, is often difficult to express on paper and so tends not be recorded. The earlier question of two circles overlapping (Fig. 2) could fall into this category for some students.

Whilst 'high insight' questions may make interesting challenges, an assessment that relies on many items that offer full marks or nothing (since little working ever appears to facilitate the award of partial credit) is less reliable as an assessment. There have to be a reasonable proportion of problem-solving items that, whilst depending on a degree of insight, lead to students revealing their strategies, so that valid, non-trivial working can be credited. The problem in Fig. 1 is an item where insight is needed, but diagram annotations or equations are likely to appear as students record parts of their thinking for their own benefit—and this can be credited.

6.6 Trialling

Each open-start problem would need to be trialled to see how students solve it (or not!). In our team we have begun to develop a 'feel' for open-start questions that 'work', but still things can go wrong, e.g. questions which are too easy or, worse, too hard. The trialling would clearly have to be done in a manner so that questions did not reach students and teachers prior to the examination. This is current practice in the development of National Curriculum tests at Key Stages 2 and 3 (tests for 11- and 14-year olds).

Trialling is also helpful in developing mark-schemes that anticipate the strategies likely to appear in the solution of a problem. Experienced examiners will be able to predict some possibilities but—given the intended novelty of the problems—may still find themselves surprised. Trialling of some kind is a way into gaining the necessary awareness. From this, mark-schemes can be written where different methods can be matched to allow the award of partial credit on an equitable basis.

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